DUU-003-016302

DUU-003-016302 Seat No. ________

M. Sc. (Maths) (Sem. III) (CBCS) Examination

May/June – 2015

Maths CMT-3002 : Functional Analysis

Faculty Code : 003
Subject Code : 016302

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all questions. Each question carries 14 marks.
(2) Figures on the right side indicate the marks alloted to each question.

1 Answer any seven questions : **2×7=14**

(i) ________ is a Banach space over $\mathbb{K}\cdot$

(A) $\left(C[0,1], \|\cdot\|_1\right)$ (B) $\left(C[0,1], \text{max.norm}\right)$

(C) $\left(C[0,1], \|\cdot\|_2\right)$ (D) $\left(C_{oo} \|\cdot\|_\infty\right)$

(ii) ________ is not a separable n.l. space over $\mathbb{K}$.

(A) $\left(l^p, \|\cdot\|_p\right),\ 1 \le P < \infty$ (B) $\left(l^\infty, \|\cdot\|_\infty\right)$

(C) $\left(C_{oo}, \|\cdot\|_\infty\right)$ (D) $\left(\mathbb{K}^n, \|\cdot\|_2\right)$

(iii) If X is a n.l space over $\mathbb{K}$ and $f \in X^1$ then $\|f\|$ = ______.

(A) $\sup\limits_{0 \ne x \in X} \frac{|f(x)|}{\|x\|}$

(B) $\sup\limits_{x \in X} |f(x)|$

(C) $\sup\limits_{0 \ne x \in X} |f(x)|$

(D) $\sup\left\{k > 0 \,|\, 1f(x)| \le k\|x\|, \forall x \in X\right\}$

(iv) ________ is Schwartz inequality.

(A) $\|x+y\| \le \|x\| + \|y\|$

(B) $|\langle x, y\rangle| \le \|x\|\,\|y\|$

(C) $\|x+y\|_p \le \|x\|_p + \|y\|_p$, $\forall x, y \in l^p$

(D) $\sum_{n=1}^{\infty} |x_n y_n| \le \|(x_1, x_2, \ldots x_n)\|_p \cdot \|(y_1, y_2, \ldots, y_n)\|_q$, for all

$(x_1, x_2, \ldots x_n, \ldots) \in l^p, (y_1, y_2, \ldots, y_{n,\ldots}) \in l^q$

(v) The co-dimension of a vector subspace Y of a vector space X is ________.

(A) $\dim X - 1$

(B) $\dim\left(X/Y\right)$

(C) $\dim Y$

(D) $\dim Y - 1$

(vi) A vector subspace Y of a n.l space X over $\mathbb{K}$ is a hyperspace iff ________.

(A) $Y = \ker f$, for some linear $f : X \to \mathbb{K}$

(B) $Y = \ker f$, for some $f \in X^1$

(C) $Y = x_o + \ker f$ for some $x_o \in X$ and linear $f : X \to \mathbb{K}$

(D) $Y = x_o + \ker f$ for some $x_o \in X$, $f \in X^1$

(vii) ________ is the statement of closed graph theorem.

(A) every bdd linear transformation is closed linear

(B) every closed linear transformation is bdd linear

(C) every closed linear transformation between two Banach space is bdd linear

(D) every onto bdd linear transformation between two Banach space is open

(viii) ____________ is a true statement.

(A) Hahn-Banach extension is unique

(B) every separable Banach space has a schauder basis

(C) weak convergence $\Rightarrow$ strong convergence in finite dimensional n.l. space

(D) In a n.l. space X over $\mathbb{K}$, $\|x+y\| = \|x\| + \|y\| \Rightarrow x, y$ are linearly dependent

(ix) ________ is not a true statement.

(A) $X \cong X'' \Rightarrow X$ is reflexive

(B) reflexive spaces are Banach spaces

(C) X' is Banch space, $\forall$ n.l spaces X

(D) a linear transformation between two n.l. spaces is bdd iff it is continuous

(x) $\|f\|_1 =$ ________, $\forall\, f \in C[0,1]$.

(A) $\max\limits_{x\in[0,1]} |f(x)|$

(B) $\int_0^1 f(t)\,dt$

(C) $\int_0^1 |f(t)|\,dt$

(D) $\int_0^1 |f(t)|^2\,dt$

2 Answer any two questions : **2×7=14**

(a) Define C_o and prove that $(C_o, \|\cdot\|_\infty)$ is a Banach space over $\mathbb{K}$ (prove only completeness).

(b) Give an example of a n.l. space over $\mathbb{K}$ which is not a Banach space with justification.

(c) True or false ? Justify. Every closed and bdd set in a finite-dimensional n.l. space over $\mathbb{K}$ is compact.

3 (a) Prove that addition, scalar multiplication and $\|\cdot\|$ are continuous operations in a n.l. space $(X, \|\cdot\|)$ over $\mathbb{K}$. **7**

(b) State and prove Riesz lemma. **7**

OR

3 (a) Prove that a finite-dimensional subspace of a n.l. space is closed. Give an example of an infinite-dimensional subspace of a n.l. space which is not closed. **7**

(b) Prove that a n.l. space X is a Banach space iff every absolutely convergent series converges in X. **7**

4 Answer any two : **2×7=14**

(a) State, without Proof, bdd inverse theorem. Can we drop the condition X, Y are Banach space in this theorem ? Justify.

(b) Prove that $\left(l^1, \|\cdot\|_1\right)' \cong \left(l^\infty, \|\cdot\|_\infty\right)$.

(c) State and prove the uniform bddness theorem.

5 Answer any two questions : **2×7=14**

(a) Define weak convergence in a n.l. space. With usual notations, prove that $l_n \xrightarrow{W} o$ in $\left(l^p, \|\cdot\|_p\right), 1 \le p < \infty$ but $\{l_n\}_{n=1}^{\infty}$ does not converge to O strongly in $\left(l^p, \|\cdot\|_p\right)$.

(b) State and prove Pythagorea's theorem in an inner product space X. Prove that for $x, y \in X$, $\|x+y\|^2 = \|x\|^2 + \|y\|^2$ does not imply x, y are othogonal.

(c) Prove that equality occurs in Schwartz inequality iff the elements are linearly dependent.

(d) Given an inner product X, a non-empty complete convex set M in X and $x \in X$, prove that $\exists$ < unique $y_o \in M$ s.t $\|x - y_o\| = d(x, M)$.
